## Jacobi and Gauss Seidal method

**Iterative methods or indirect methods:**

We start with approximation to the true solution and by applying the method repeatedly we get better and better approximation till accurate solution is achieved.

There are two iterative methods for solving simultaneous equations:

- Jacobi's Method ( Method of simultaneous correction )
- Gauss-Seidal Method ( Method of successive correction )

**Jacobi's Method**

The method is illustrated by taking an example.

Let

$$\left.\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= b_1 \\ a_{21}x + a_{22}y + a_{23}z &= b_2 \\ a_{31}x + a_{32}y + a_{33}z &= b_3 \end{aligned}\right\} \quad \text{...(1)}$$

After division by suitable constants and transposition, the equations can be written as

$$\left.\begin{aligned} x &= c_1 - k_{12}y - k_{13}z \\ y &= c_2 - k_{21}x - k_{23}z \\ z &= c_3 - k_{31}x - k_{32}y \end{aligned}\right\} \quad \text{...(2)}$$

Let us assume $x = 0$, $y = 0$ and $z = 0$ as first approximation, substituting the values of $x, y, z$ on the right hand side of (2), we get $x = c_1$, $y = c_2$, $z = c_3$. This is the second approximation to the solution of the equations.

Again substituting these values of $x, y, z$ in (2) we get a third approximation.

The process is repeated till two successive approximations are equal or nearly equal.

**Note:** Condition for using the iterative methods is that the coefficients in the leading diagonal are large compared to the other. If are not so, then on interchanging the equation we can make the leading diagonal dominant diagonal.

Example: Solve by Jacobi's method

$$4x + y + 3z = 17$$
$$x + 5y + z = 14$$
$$2x - y + 8z = 12$$

**Solution.** The above equations can be written as

$$\left.\begin{aligned} x &= \frac{17}{4} - \frac{y}{4} - \frac{3z}{4} \\ y &= \frac{14}{5} - \frac{x}{5} - \frac{z}{5} \\ z &= \frac{3}{2} - \frac{x}{4} + \frac{y}{8} \end{aligned}\right] \quad \ldots(1)$$

On substituting $x = y = z = 0$ on the right hand side of (1), we get

$$x = \frac{17}{4}, \quad y = \frac{14}{5}, \quad z = \frac{3}{2}$$

Again substituting these values of x, y, z on R.H.S. of (1), we obtain

$$x = \frac{17}{4} - \frac{7}{10} - \frac{9}{8} = \frac{97}{40}$$
$$y = \frac{14}{5} - \frac{17}{20} - \frac{3}{10} = \frac{33}{20}$$
$$z = \frac{3}{2} - \frac{17}{16} + \frac{7}{20} = \frac{63}{80}$$

Again putting these values on R.H.S. of (1) we get next approximations.

$$x = \frac{17}{4} - \frac{33}{80} - \frac{189}{320} = \frac{1039}{320} = 3.25$$
$$y = \frac{14}{5} - \frac{97}{200} - \frac{63}{400} = \frac{863}{400} = 2.16$$
$$z = \frac{3}{2} - \frac{97}{160} + \frac{33}{160} = \frac{176}{160} = 1.1$$

Substituting, again, the values of $x, y, z$ on R.H.S. of (1) we get

$$x = \frac{17}{4} - \frac{2.16}{4} - \frac{3\,(1.1)}{4} = 2.885$$

$$y = \frac{14}{5} - \frac{3.25}{5} - \frac{1.1}{5} = 1.93$$

$$z = \frac{3}{2} - \frac{3.25}{4} + \frac{2.16}{8} = 0.96$$

Repeating the process for x = 2.885, y = 1.93, z = 0.96 we have

$$x = \frac{17}{4} - \frac{1.93}{4} - \frac{3}{4} \times 0.96 = 4.25 - 0.48 - 0.72 = 3.05$$

$$y = \frac{14}{5} - \frac{2.885}{5} - \frac{0.96}{5} = 2.8 - 0.577 - 0.192 = 2.03$$

$$z = \frac{3}{2} - \frac{2.885}{4} + \frac{1.93}{8} = 1.5 - 0.721 + 0.241 = 1.02$$

This can be written in a table

| *Iterations* | *1* | *2* | *3* | *4* | *5* | *6* |
|---|---|---|---|---|---|---|
| $x = \frac{17}{4} - \frac{y}{4} - \frac{3z}{4}$ | 0 | $\frac{17}{4} = 4.25$ | $\frac{97}{40} = 2.425$ | $\frac{1039}{320} = 3.25$ | 2.885 | 3.05 |
| $y = \frac{14}{5} - \frac{x}{5} - \frac{z}{5}$ | 0 | $\frac{14}{5} = 2.8$ | $\frac{33}{20} = 1.65$ | $\frac{863}{400} = 2.16$ | 1.93 | 2.03 |
| $z = \frac{3}{2} - \frac{x}{4} + \frac{y}{8}$ | 0 | $\frac{3}{2} = 1.5$ | $\frac{63}{80} = 0.7875$ | $\frac{176}{160} = 1.1$ | 0.96 | 1.02 |

After 6th iteration

$$x = 3.05, \;\; y = 2.03, \;\; z = 1.02.$$

The actual values are

$$x = 3, \;\; y = 2, \;\; z = 1.$$

Ans.

Gauss- Seidel method:

Gauss- Seidel method is modification of Jacobi's method. In place of substituting the same set of values in all the three equations, we use in each step the value obtained in the earlier step.

Step 1: We put y = 0 and z = 0 in first equation of (2) and we get x = c1. Then in second equation we put x = c1 and z = 0 and obtain y. In the third equation we use the values of x and y obtained earlier and gets z.

Step 2: We repeat the above procedure. In the first equation we put the values of y and z obtained in step 1 and re-determine x. By using the new value of x and value of z obtained in step 1 we re-determine y and so on.

Remember: The latest values of the unknowns are used in each step.

The method is illustrated by taking an example.

Let

$$\left.\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= b_1 \\ a_{21}x + a_{22}y + a_{23}z &= b_2 \\ a_{31}x + a_{32}y + a_{33}z &= b_3 \end{aligned}\right\} \quad \text{...(1)}$$

After division by suitable constants and transposition, the equtions can be written as

$$\left.\begin{aligned} x &= c_1 - k_{12}y - k_{13}z \\ y &= c_2 - k_{21}x - k_{23}z \\ z &= c_3 - k_{31}x - k_{32}y \end{aligned}\right\} \quad \text{...(2)}$$